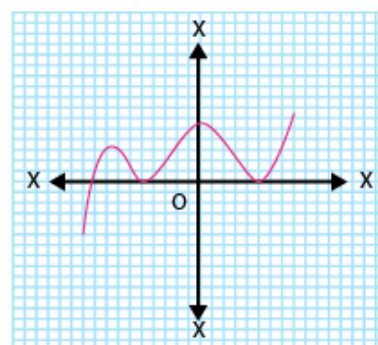
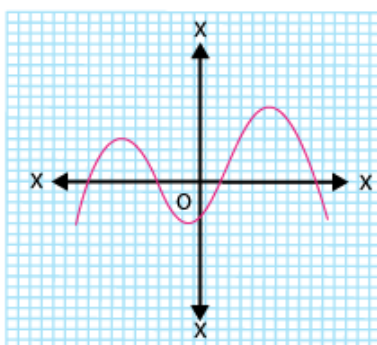
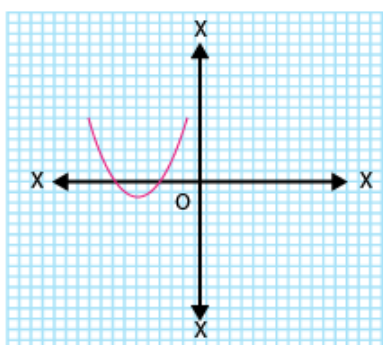
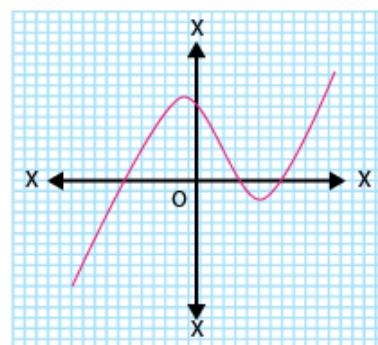
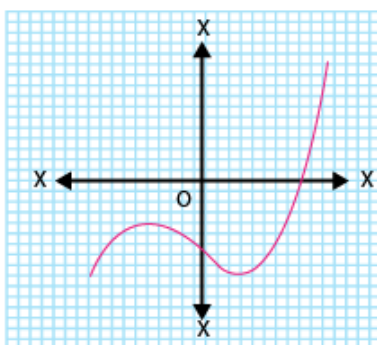
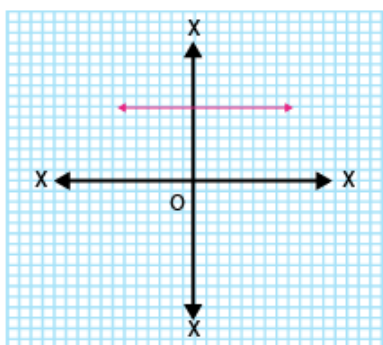


## CHAPTER 2 – POLYNOMIALS

### CLASS 10 NCERT SOLUTION

#### Exercise 2.1 Page: 28

1. The graphs of  $y = p(x)$  are given in Fig. 2.10 below, for some polynomials  $p(x)$ . Find the number of zeroes of  $p(x)$ , in each case.



#### Solutions:

##### Graphical method to find zeroes:-

Total number of zeroes in any polynomial equation = total number of times the curve intersects x-axis.

(i) In the given graph, the number of zeroes of  $p(x)$  is 0 because the graph is parallel to x-axis does not cut it at any point.

(ii) In the given graph, the number of zeroes of  $p(x)$  is 1 because the graph intersects the x-axis at only one point.

(iii) In the given graph, the number of zeroes of  $p(x)$  is 3 because the graph intersects the x-axis at any three points.

(iv) In the given graph, the number of zeroes of  $p(x)$  is 2 because the graph intersects the x-axis at two points.

(v) In the given graph, the number of zeroes of  $p(x)$  is 4 because the graph intersects the x-axis at four points.

(vi) In the given graph, the number of zeroes of  $p(x)$  is 3 because the graph intersects the x-axis at three points.

Exercise 2.2 Page: 33

**1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.**

**Solutions:**

**(i)  $x^2 - 2x - 8$**

$$\Rightarrow x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$$

Therefore, zeroes of polynomial equation  $x^2 - 2x - 8$  are (4, -2)

$$\text{Sum of zeroes} = 4 - 2 = 2 = -(-2)/1 = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = 4 \times (-2) = -8 = -(8)/1 = (\text{Constant term})/(\text{Coefficient of } x^2)$$

**(ii)  $4s^2 - 4s + 1$**

$$\Rightarrow 4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) = (2s-1)(2s-1)$$

Therefore, zeroes of polynomial equation  $4s^2 - 4s + 1$  are (1/2, 1/2)

$$\text{Sum of zeroes} = (1/2) + (1/2) = 1 = -4/4 = -(\text{Coefficient of } s)/(\text{Coefficient of } s^2)$$

$$\text{Product of zeroes} = (1/2) \times (1/2) = 1/4 = (\text{Constant term})/(\text{Coefficient of } s^2)$$

**(iii)  $6x^2 - 3 - 7x$**

$$\Rightarrow 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$$

Therefore, zeroes of polynomial equation  $6x^2 - 3 - 7x$  are (-1/3, 3/2)

$$\text{Sum of zeroes} = -(1/3) + (3/2) = (7/6) = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

Product of zeroes =  $-(1/3) \times (3/2) = -(3/6) = (\text{Constant term}) / (\text{Coefficient of } x^2)$

**(iv)  $4u^2+8u$**

$$\Rightarrow 4u(u+2)$$

Therefore, zeroes of polynomial equation  $4u^2 + 8u$  are  $(0, -2)$ .

Sum of zeroes =  $0+(-2) = -2 = -(8/4) = -(\text{Coefficient of } u) / (\text{Coefficient of } u^2)$

Product of zeroes =  $0 \times -2 = 0 = 0/4 = (\text{Constant term}) / (\text{Coefficient of } u^2)$

**(v)  $t^2-15$**

$$\Rightarrow t^2 = 15 \text{ or } t = \pm\sqrt{15}$$

Therefore, zeroes of polynomial equation  $t^2 - 15$  are  $(\sqrt{15}, -\sqrt{15})$

Sum of zeroes =  $\sqrt{15} + (-\sqrt{15}) = 0 = -(0/1) = -(\text{Coefficient of } t) / (\text{Coefficient of } t^2)$

Product of zeroes =  $\sqrt{15} \times (-\sqrt{15}) = -15 = -15/1 = (\text{Constant term}) / (\text{Coefficient of } t^2)$

**(vi)  $3x^2-x-4$**

$$\Rightarrow 3x^2-4x+3x-4 = x(3x-4)+1(3x-4) = (3x - 4)(x + 1)$$

Therefore, zeroes of polynomial equation  $3x^2 - x - 4$  are  $(4/3, -1)$

Sum of zeroes =  $(4/3)+(-1) = (1/3) = -(-1/3) = -(\text{Coefficient of } x) / (\text{Coefficient of } x^2)$

Product of zeroes =  $(4/3) \times (-1) = (-4/3) = (\text{Constant term}) / (\text{Coefficient of } x^2)$

**2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.**

**(i)  $1/4, -1$**

**Solution:**

From the formulas of sum and product of zeroes, we know,

$$\text{Sum of zeroes} = \alpha + \beta$$

$$\text{Product of zeroes} = \alpha \beta$$

$$\text{Sum of zeroes} = \alpha + \beta = 1/4$$

$$\text{Product of zeroes} = \alpha \beta = -1$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (1/4)x + (-1) = 0$$

$$4x^2 - x - 4 = 0$$

**Thus,  $4x^2 - x - 4$  is the quadratic polynomial.**

**(ii)  $\sqrt{2}, 1/3$**

**Solution:**

$$\text{Sum of zeroes} = \alpha + \beta = \sqrt{2}$$

$$\text{Product of zeroes} = \alpha\beta = 1/3$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (\sqrt{2})x + (1/3) = 0$$

$$3x^2 - 3\sqrt{2}x + 1 = 0$$

**Thus,  $3x^2 - 3\sqrt{2}x + 1$  is the quadratic polynomial.**

**(iii)  $0, \sqrt{5}$**

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 0$$

$$\text{Product of zeroes} = \alpha\beta = \sqrt{5}$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly

as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (0)x + \sqrt{5} = 0$$

**Thus,  $x^2 + \sqrt{5}$  is the quadratic polynomial.**

**(iv)  $1, 1$**

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 1$$

$$\text{Product of zeroes} = \alpha \beta = 1$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - x + 1 = 0$$

**Thus,  $x^2 - x + 1$  is the quadratic polynomial.**

**(v)  $-1/4, 1/4$**

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta = -1/4$$

$$\text{Product of zeroes} = \alpha \beta = 1/4$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (-1/4)x + (1/4) = 0$$

$$4x^2 + x + 1 = 0$$

**Thus,  $4x^2 + x + 1$  is the quadratic polynomial.**

**(vi) 4, 1**

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta =$$

$$\text{Product of zeroes} = \alpha\beta = 1$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - 4x + 1 = 0$$

Thus,  $x^2 - 4x + 1$  is the quadratic polynomial.

Exercise 2.3 Page: 36

**1. Divide the polynomial  $p(x)$  by the polynomial  $g(x)$  and find the quotient and remainder in each of the following:**

**(i)  $p(x) = x^3 - 3x^2 + 5x - 3$  ,  $g(x) = x^2 - 2$**

**Solution:**

Given,

Dividend =  $p(x) = x^3 - 3x^2 + 5x - 3$

Divisor =  $g(x) = x^2 - 2$

$$\begin{array}{r} x - 3 \\ x^2 - 2 \overline{) x^3 - 3x^2 + 5x - 3} \\ \underline{-} \phantom{x^3} \\ x^3 + 0x^2 - 2x \\ \underline{-} \phantom{x^3} \\ -3x^2 + 7x - 3 \\ \phantom{-} \underline{-} \phantom{x^3} \\ -3x^2 + 0x + 6 \\ \phantom{-} \phantom{x^3} \phantom{-} \underline{-} \phantom{x^3} \\ 7x - 9 \end{array}$$

Therefore, upon division we get,

Quotient =  $x - 3$

Remainder =  $7x - 9$

**(ii)  $p(x) = x^4 - 3x^2 + 4x + 5$  ,  $g(x) = x^2 + 1 - x$**

**Solution:**

Given,

Dividend =  $p(x) = x^4 - 3x^2 + 4x + 5$

$$\text{Divisor} = g(x) = x^2 + 1 - x$$

$$\begin{array}{r}
 \phantom{x^2 - x + 1} \quad \quad \quad x^2 \quad +x \quad -3 \\
 \phantom{x^2 - x + 1} \quad \quad \quad \hline
 x^2 - x + 1 \quad \quad \quad \left) \begin{array}{l} x^4 + 0x^3 - 3x^2 + 4x + 5 \\ \\ - \\ x^4 - x^3 + x^2 \\ \hline \phantom{x^4 - x^3} x^3 - 4x^2 + 4x + 5 \\ \\ - \\ \phantom{x^4 - x^3} x^3 - x^2 + x \\ \hline \phantom{x^4 - x^3} \phantom{x^3} - 3x^2 + 3x + 5 \\ \\ - \\ \phantom{x^4 - x^3} \phantom{x^3} - 3x^2 + 3x - 3 \\ \hline \phantom{x^4 - x^3} \phantom{x^3} \phantom{- 3x^2} + 3x - 3 \\ \hline \phantom{x^4 - x^3} \phantom{x^3} \phantom{- 3x^2} \phantom{+ 3x} - 3 \\ \hline \phantom{x^4 - x^3} \phantom{x^3} \phantom{- 3x^2} \phantom{+ 3x} \phantom{- 3} 8 \end{array}
 \end{array}$$

Therefore, upon division we get,

$$\text{Quotient} = x^2 + x - 3$$

$$\text{Remainder} = 8$$

**(iii)  $p(x) = x^4 - 5x + 6$ ,  $g(x) = 2 - x^2$**

**Solution:**

Given,

$$\text{Dividend} = p(x) = x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6$$

$$\text{Divisor} = g(x) = 2 - x^2 = -x^2 + 2$$

$$\begin{array}{r}
 -x^2 + 2 \quad \overline{) \quad -x^2 - 2} \\
 \underline{-x^4 + 0x^3 + 0x^2 - 5x + 6} \\
 x^4 + 0x^3 - 2x^2 \\
 \underline{\phantom{x^4} + 2x^2 - 5x + 6} \\
 \phantom{x^4} - 2x^2 - 5x + 6 \\
 \underline{\phantom{x^4} + 2x^2 + 0x - 4} \\
 \phantom{x^4} - 5x + 10
 \end{array}$$

Therefore, upon division we get,

$$\text{Quotient} = -x^2 - 2$$

$$\text{Remainder} = -5x + 10$$

**2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:**

**(i)  $t^2 - 3$ ,  $2t^4 + 3t^3 - 2t^2 - 9t - 12$**

**Solutions:**

Given,

$$\text{First polynomial} = t^2 - 3$$

$$\text{Second polynomial} = 2t^4 + 3t^3 - 2t^2 - 9t - 12$$



$$\begin{array}{r}
 \phantom{t^2 - 3} \quad 2t^2 + 3t + 4 \\
 t^2 - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{-} \\
 2t^4 + 0t^3 - 6t^2 \\
 \underline{-} \\
 \phantom{2t^4} 3t^3 + 4t^2 - 9t - 12 \\
 \phantom{2t^4} \underline{-} \\
 \phantom{2t^4} 3t^3 + 0t^2 - 9t \\
 \phantom{2t^4} \underline{-} \\
 \phantom{2t^4} \phantom{3t^3} 4t^2 + 0t - 12 \\
 \phantom{2t^4} \phantom{3t^3} \underline{-} \\
 \phantom{2t^4} \phantom{3t^3} 4t^2 + 0t - 12 \\
 \phantom{2t^4} \phantom{3t^3} \underline{-} \\
 \phantom{2t^4} \phantom{3t^3} \phantom{4t^2} 0
 \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that,  $t^2-3$  is a factor of  $2t^2+3t+4$ .

**(ii)**  $x^2+3x+1$  ,  $3x^4+5x^3-7x^2+2x+2$

**Solutions:**

Given,

First polynomial =  $x^2+3x+1$

Second polynomial =  $3x^4+5x^3-7x^2+2x+2$

$$\begin{array}{r}
 x^2 + 3x + 1 \quad \overline{) \quad 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \phantom{+ 2x + 2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \phantom{+ 2} \\
 2x^2 + 6x + 2 \\
 \underline{2x^2 + 6x + 2} \\
 0
 \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that,  $x^2 + 3x + 1$  is a factor of  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ .

(iii)  $x^3 - 3x + 1$ ,  $x^5 - 4x^3 + x^2 + 3x + 1$

**Solutions:**

Given,

First polynomial =  $x^3 - 3x + 1$

Second polynomial =  $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 x^3 - 3x + 1 \quad \overline{) \quad x^5 + 0x^4 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 + 0x^4 - 3x^3 + x^2} \phantom{+ 3x + 1} \\
 -x^3 + 0x^2 + 3x + 1 \\
 \underline{-x^3 + 0x^2 + 3x - 1} \\
 2
 \end{array}$$

As we can see, the remainder is not equal to 0. Therefore, we say that,  $x^3-3x+1$  is not a factor of  $x^5-4x^3+x^2+3x+1$ .

**3. Obtain all other zeroes of  $3x^4+6x^3-2x^2-10x-5$ , if two of its zeroes are  $\sqrt{5/3}$  and  $-\sqrt{5/3}$ .**

**Solutions:**

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$\sqrt{5/3}$  and  $-\sqrt{5/3}$  are zeroes of polynomial  $f(x)$ .

$$\square (x - \sqrt{5/3})(x + \sqrt{5/3}) = x^2 - (5/3) = 0$$

$(3x^2-5)=0$ , is a factor of given polynomial  $f(x)$ .

Now, when we will divide  $f(x)$  by  $(3x^2-5)$  the quotient obtained will also be a factor of  $f(x)$  and the remainder will be 0.

$$\begin{array}{r}
 x^2 + 2x + 1 \\
 3x^2 - 5 \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\
 \underline{3x^4 \quad - 5x^2} \phantom{- 10x - 5} \\
 (-) \phantom{3x^4} \quad (+) \phantom{- 5x^2} \\
 \hline
 + 6x^3 + 3x^2 - 10x - 5 \\
 \underline{- 6x^3 \quad - 10x} \\
 (+) \phantom{6x^3} \quad (-) \phantom{- 10x} \\
 \hline
 \phantom{6x^3} \quad 3x^2 \quad - 5 \\
 \phantom{6x^3} \quad 3x^2 \quad - 5 \\
 \hline
 (-) \phantom{6x^3} \quad (+) \phantom{- 5} \\
 \hline
 \phantom{6x^3} \quad \phantom{3x^2} \quad 0 \\
 \hline
 \hline
 \phantom{6x^3} \quad \phantom{3x^2} \quad 0
 \end{array}$$

Therefore,  $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$

Now, on further factorizing  $(x^2 + 2x + 1)$  we get,

$$x^2 + 2x + 1 = x^2 + x + x + 1 = 0$$

$$x(x+1)+1(x+1) = 0$$

$$(x+1)(x+1) = 0$$

So, its zeroes are given by:  $x = -1$  and  $x = -1$ .

Therefore, all four zeroes of given polynomial equation are:

$$\sqrt{(5/3)}, -\sqrt{(5/3)}, -1 \text{ and } -1.$$

Hence, is the answer.

**4. On dividing  $x^3-3x^2+x+2$  by a polynomial  $g(x)$ , the quotient and remainder were  $x-2$  and  $-2x+4$ , respectively. Find  $g(x)$ .**

**Solution:**

Given,

$$\text{Dividend, } p(x) = x^3-3x^2+x+2$$

$$\text{Quotient} = x-2$$

$$\text{Remainder} = -2x+4$$

We have to find the value of Divisor,  $g(x) = ?$

As we know,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\therefore x^3-3x^2+x+2 = g(x) \times (x-2) + (-2x+4)$$

$$x^3-3x^2+x+2 - (-2x+4) = g(x) \times (x-2)$$

$$\text{Therefore, } g(x) \times (x-2) = x^3-3x^2+x+2$$

Now, for finding  $g(x)$  we will divide  $x^3-3x^2+x+2$  with  $(x-2)$

$$\begin{array}{r}
 x^2 - x + 1 \\
 x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2} \phantom{+ 3x - 2} \\
 (-) (+) \\
 \phantom{x^3 - } - x^2 + 3x - 2 \\
 \phantom{x^3 - } \underline{-x^2 + 2x} \phantom{- 2} \\
 (+) (-) \\
 \phantom{x^3 - } \phantom{-x^2 + } x - 2 \\
 \phantom{x^3 - } \phantom{-x^2 + } \underline{x - 2} \\
 (-) (+) \\
 \phantom{x^3 - } \phantom{-x^2 + } \phantom{x - } 0
 \end{array}$$

Therefore,  $g(x) = (x^2 - x + 1)$

**5. Give examples of polynomials  $p(x)$ ,  $g(x)$ ,  $q(x)$  and  $r(x)$ , which satisfy the division algorithm and**

**(i)  $\deg p(x) = \deg q(x)$**

**(ii)  $\deg q(x) = \deg r(x)$**

**(iii)  $\deg r(x) = 0$**

**Solutions:**

According to the division algorithm, dividend  $p(x)$  and divisor  $g(x)$  are two polynomials, where  $g(x) \neq 0$ . Then we can find the value of quotient  $q(x)$  and remainder  $r(x)$ , with the help of below given formula;

Dividend = Divisor  $\times$  Quotient + Remainder

$\therefore p(x) = g(x) \times q(x) + r(x)$

Where  $r(x) = 0$  or degree of  $r(x) <$  degree of  $g(x)$ .

Now let us proof the three given cases as per division algorithm by taking examples for each.

**(i)  $\deg p(x) = \deg q(x)$**

Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.

Let us take an example,  $3x^2 + 3x + 3$  is a polynomial to be divided by 3.

$$\text{So, } (3x^2+3x+3)/3 = x^2+x+1 = q(x)$$

Thus, you can see, the degree of quotient is equal to the degree of dividend.

Hence, division algorithm is satisfied here.

**(ii)  $\deg q(x) = \deg r(x)$**

Let us take an example ,  $p(x)=x^2+x$  is a polynomial to be divided by  $g(x)=x$ .

$$\text{So, } (x^2+x)/x = x+1 = q(x)$$

$$\text{Also, remainder, } r(x) = 0$$

Thus, you can see, the degree of quotient is equal to the degree of remainder.

Hence, division algorithm is satisfied here.

**(iii)  $\deg r(x) = 0$**

The degree of remainder is 0 only when the remainder left after division algorithm is constant.

Let us take an example,  $p(x) = x^2+1$  is a polynomial to be divided by  $g(x)=x$ .

$$\text{So, } (x^2+1)/x = x = q(x)$$

$$\text{And } r(x)=1$$

Clearly, the degree of remainder here is 0.

Hence, division algorithm is satisfied here.

Exercise 2.4 Page: 36

**1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:**

**(i)  $2x^3+x^2-5x+2$ ;  $-1/2, 1, -2$**

**Solution:**

$$\text{Given, } p(x) = 2x^3+x^2-5x+2$$

$$\text{And zeroes for } p(x) \text{ are } = 1/2, 1, -2$$

$$\therefore p(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2 = (1/4) + (1/4) - (5/2) + 2 = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0$$

Hence, proved  $1/2, 1, -2$  are the zeroes of  $2x^3 + x^2 - 5x + 2$ .

Now, comparing the given polynomial with general expression, we get;

$$\therefore ax^3 + bx^2 + cx + d = 2x^3 + x^2 - 5x + 2$$

$$a=2, b=1, c=-5 \text{ and } d=2$$

As we know, if  $\alpha, \beta, \gamma$  are the zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$ , then;

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a.$$

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -1/2 = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (1/2 \times 1) + (1 \times -2) + (-2 \times 1/2) = -5/2 = c/a$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = -2/2 = -d/a$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

**(ii)  $x^3 - 4x^2 + 5x - 2$ ; 2, 1, 1**

**Solution:**

$$\text{Given, } p(x) = x^3 - 4x^2 + 5x - 2$$

And zeroes for  $p(x)$  are 2, 1, 1.

$$\therefore p(2) = 2^3 - 4(2)^2 + 5(2) - 2 = 0$$

$$p(1) = 1^3 - (4 \times 1^2) + (5 \times 1) - 2 = 0$$

Hence proved, 2, 1, 1 are the zeroes of  $x^3 - 4x^2 + 5x - 2$

Now, comparing the given polynomial with general expression, we get;

$$\therefore ax^3 + bx^2 + cx + d = x^3 - 4x^2 + 5x - 2$$

$$a = 1, b = -4, c = 5 \text{ and } d = -2$$

As we know, if  $\alpha, \beta, \gamma$  are the zeroes of the cubic polynomial  $ax^3+bx^2+cx+d$ , then;

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha \beta \gamma = -d/a.$$

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = 2+1+1 = 4 = -(-4)/1 = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 \times 1 + 1 \times 1 + 1 \times 2 = 5 = 5/1 = c/a$$

$$\alpha\beta\gamma = 2 \times 1 \times 1 = 2 = -(-2)/1 = -d/a$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

**2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.**

**Solution:**

Let us consider the cubic polynomial is  $ax^3+bx^2+cx+d$  and the values of the zeroes of the polynomials be  $\alpha, \beta, \gamma$ .

As per the given question,

$$\alpha + \beta + \gamma = -b/a = 2/1$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a = -7/1$$

$$\alpha \beta \gamma = -d/a = -14/1$$

Thus, from above three expressions we get the values of coefficient of polynomial.

$$a = 1, b = -2, c = -7, d = 14$$

Hence, the cubic polynomial is  $x^3-2x^2-7x+14$

**3. If the zeroes of the polynomial  $x^3-3x^2+x+1$  are  $a - b, a, a + b$ , find  $a$  and  $b$ .**

**Solution:**

We are given with the polynomial here,

$$p(x) = x^3-3x^2+x+1$$



And zeroes are given as  $a - b$ ,  $a$ ,  $a + b$

Now, comparing the given polynomial with general expression, we get;

$$\therefore px^3 + qx^2 + rx + s = x^3 - 3x^2 + x + 1$$

$$p = 1, q = -3, r = 1 \text{ and } s = 1$$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$-q/p = 3a$$

Putting the values  $q$  and  $p$ .

$$-(-3)/1 = 3a$$

$$a = 1$$

Thus, the zeroes are  $1 - b$ ,  $1$ ,  $1 + b$ .

$$\text{Now, product of zeroes} = 1(1 - b)(1 + b)$$

$$-s/p = 1 - b^2$$

$$-1/1 = 1 - b^2$$

$$b^2 = 1 + 1 = 2$$

$$b = \sqrt{2}$$

Hence,  $1 - \sqrt{2}$ ,  $1$ ,  $1 + \sqrt{2}$  are the zeroes of  $x^3 - 3x^2 + x + 1$ .

**4. If two zeroes of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 \pm \sqrt{3}$ , find other zeroes.**

**Solution:**

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$$\text{Let } f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$$

Since  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$  are zeroes of given polynomial  $f(x)$ .

$$\therefore [x - (2 + \sqrt{3})] [x - (2 - \sqrt{3})] = 0$$

$$(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = 0$$

On multiplying the above equation we get,

$x^2 - 4x + 1$ , this is a factor of a given polynomial  $f(x)$ .

Now, if we will divide  $f(x)$  by  $g(x)$ , the quotient will also be a factor of  $f(x)$  and the remainder will be 0.

$$\begin{array}{r}
 x^2 - 2x - 35 \\
 x^2 - 4x + 1 \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 \underline{x^4 - 4x^3 + x^2} \phantom{- 35} \\
 (-) \quad (+) \quad (-) \\
 -2x^3 - 27x^2 + 138x - 35 \\
 \underline{-2x^3 + 8x^2 - 2x} \phantom{- 35} \\
 (+) \quad (-) \quad (+) \\
 -35x^2 + 140x - 35 \\
 \underline{-35x^2 + 140x - 35} \\
 (+) \quad (-) \quad (+) \\
 0
 \end{array}$$

So,  $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now, on further factorizing  $(x^2 - 2x - 35)$  we get,

$$x^2 - (7-5)x - 35 = x^2 - 7x + 5x + 35 = 0$$

$$x(x - 7) + 5(x - 7) = 0$$

$$(x+5)(x-7) = 0$$

So, its zeroes are given by:

$$x = -5 \text{ and } x = 7.$$

Therefore, all four zeroes of given polynomial equation are:  $2 + \sqrt{3}$ ,  $2 - \sqrt{3}$ ,  $-5$  and  $7$ .